



$\mathbb{Z}^n$  is a free  $\mathbb{Z}$ -module of rank  $n$ . Let  $\mathcal{B}$  be a basis for  $\mathbb{Z}^n$ . Then  $\mathbb{Z}^n \cong \mathbb{Z} \oplus \mathbb{Z} \oplus \dots \oplus \mathbb{Z}$  ( $n$  times).

Let  $M$  be a finitely generated  $\mathbb{Z}$ -module. Then  $M \cong \mathbb{Z}^r \oplus \mathbb{Z}/d_1\mathbb{Z} \oplus \dots \oplus \mathbb{Z}/d_k\mathbb{Z}$ , where  $r$  is the rank of  $M$  and  $d_1, \dots, d_k$  are the invariant factors of  $M$ .

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